Slotted ALOHA in Small Cell Networks: How to Design Codes on Random Geometric Graphs?

Dejan Vukobratović
Associate Professor,
DEET-UNS
University of Novi Sad, Serbia

Joint work with
Dragana Bajović and Dušan Jakovetić

DLR/TUM Workshop, Munich, 26.07.2016
Motivation: Machine-Type Communications (MTC) in Future 5G Small Cell Networks

Small Base Station

Sensor Node
Problem: Massive Uncoordinated Random Access in Ultra-Dense Scenario
Approach: Slotted ALOHA with Interference Cancellation Interpreted as Codes on Graphs
Outline

- Single Base-Station Model
  - Slotted ALOHA w SIC
  - LDPC Codes

- Multiple Base-Station Model
  - Cooperative Slotted ALOHA
  - Codes on Random Geometric Graphs

- Ongoing/Future Work
Outline

- Single Base-Station Model
  - Slotted ALOHA w SIC
  - LDPC Codes

- Multiple Base-Station Model
  - Cooperative Slotted ALOHA
  - Codes on Random Geometric Graphs

- Ongoing/Future Work
Slotted ALOHA

SA protocol

- Users access slots with slot-access probability $p$
- Average slot load $G = p \cdot n$
- Idle slots are waste
- Singletons are useful
- Collisions are destructive

Throughput:

Average fraction of singletons: $T = Ge^{-G}$

$T_{max} = \frac{1}{e} \approx 0.37 \ (\text{when } G = 1)$

Framed Slotted ALOHA

FSA protocol

- Slots are organized in frames
- If a user has a packet to send, it will send in upcoming frame in a randomly selected slot
- Average load is $G = \frac{n}{\tau}$

Throughput:
- Average fraction of singletons: $T = Ge^{-G}$
- $T_{max} = \frac{1}{e} \approx 0.37$ (when $G = 1$)

Collision Resolution Diversity Slotted ALOHA

CRD-SA protocol

- Users repeat transmissions in multiple slots
  - Repetition information in packet header
  - Same number of repetitions per user

- Collisions can be exploited

- Iterative interference cancellation across slots
  - Can be stuck in a stopping set!

- Throughput:
  \[ T \approx 0.55 \] for CRDSA with two repetitions per user

---

Irregular Repetition Slotted ALOHA

IRSA protocol

- Iterative interference cancellation equivalent to iterative erasure decoding of LDPC codes
- Improved design (generalization of CRDSA)
  - No. of repetitions varies across users
  - Every user selects its no. of repeated transmissions (degree $d$) according to a predefined degree distribution $\Lambda_d$
- There exists an asymptotic threshold load $G^*$ below which probability user is collected $\rightarrow 1$
  - $H^* \sim 0.97$

Frameless ALOHA

- Idea: Apply paradigm of rateless codes
- No predefined frame length
  - Slots are successively added until sufficiently many users are resolved
- Optimization of the slot degree distribution
  - Implicitly controlled through user behavior
    - slot access probability $p$

Modeled as LDPC codes for erasure channels

Goal: Max Throughput: $T = GP_{\text{dec}}$

Decoding Probability Analysis

- Asymptotic analysis
  - Density Evolution
- Finite-Length analysis
  - Stopping Sets

Outline

- Single Base-Station Model
  - Slotted ALOHA w SIC
  - LDPC Codes

- Multiple Base-Station Model
  - Cooperative Slotted ALOHA
  - Codes on Random Geometric Graphs

- Ongoing/Future Work
Slotted ALOHA with Multiple Base Stations (SA-MBS)

Small Base Station

Sensor Node
SA-MBS system model

Base station deployment, user locations

$n$ users/devices, $m$ base stations...

...deployed independently uniformly at random (PPP) over unit square area.
SA-MBS system model

Transmission protocol

- Run **frame-slotted ALOHA** in parallel across all BS
  - $\tau$ slots per frame – slot synchronized across all base stations
  - User may be active (send packet replica) in several slots per frame
  - User is heard by all base stations that cover it

- Signal at the base station $j$ at slot $t$:
  - sum of signals of all users active at slot $t$
  - covered by the base station $j$
User collection

- Base station “collects” a user whenever it detects a “clean” signal

A user is collected if it is collected by any base station!
Asymptotic analysis

Asymptotic setup
- \( n, m(n), \tau(n) \rightarrow \infty \) and \( r(n) \rightarrow 0 \)
- \( \delta, G > 0 \), where \( \delta = r^2 \pi \cdot m \) and \( G = n/(m\tau) \)

Metrics of interest
- Probability of user collection:
  \[
P(U_i \text{ coll.}) = E \left[ \frac{1}{n} \sum_{i=1}^{n} I\{U_i \text{ coll.}\} \right]
  \]
  - Upper bounded by user coverage probability \( 1 - e^{-\delta} \)

- Normalized throughput:
  \[
  T(G) = \frac{1}{m\tau} E[\sum_{i=1}^{n} I\{U_i \text{ coll.}\}] = G \cdot P(U_i \text{ coll.})
  \]

- Threshold Load: \( G^*(\delta) = \sup\{G \geq 0 : P(U_i \text{ coll.}) \rightarrow 1 - e^{-\delta}\} \)
Multiple Base Station Model: No Cooperation

Slotted ALOHA in Multi-Base Station (SA-MBS) w/o Cooperation

- Base stations do not cooperate
- Ordinary framed SA (no SIC in time-domain)
- **Throughput?**
Multi-Base Station Model: Decoding via Spatial Cooperation

Slotted ALOHA in Multi-Base Station (SA-MBS) w Spatial Cooperation

- Base stations who share the same users do cooperate
- SA with SIC in spatial-domain (after each time slot)
- **Erasure Decoding on Random Bipartite Geometric Graph**
Multi-Base Station Model:
Decoding via Spatial Cooperation

Spatial Cooperation decoding algorithm

One iteration at arbitrary base station after each slot $t$

1) **Check signal**: BS $j$ checks whether its received signal $y_{j,t}$ corresponds to a singleton; If yes, it performs Collect & Transmit step, otherwise it performs Receive & Update step

2) **Collect & Transmit**: BS $j$ collects a user $u$ and transmits $x_u$ to all BS $k$ adjacent to user $u$ (this is known to BS in advance). BS $j$ leaves the algorithm.

3) **Receive & Update**: BS $j$ scans all the received messages from its neighbors and identifies distinct set of user signals $x_u$. Then it removes all the signals from this set from $y_{j,t}$ and goes to step one in the next iteration

**Fully Distributed**: base stations communicate only with neighboring base stations!
Main results

Spatial Cooperation:

- **[Upper Bound on \( P(U_i \text{ coll.}) \):]**
  \[
  P(U_i \text{ coll.}) \leq 1 - e^{-\delta} - (1 - e^{-\delta/4})e^{-2\delta}(1 - e^{-G\delta/4})
  \]

- **[Threshold Load]:**
  \[
  G^* (\delta) = 0
  \]
  The probability \( P(U_i \text{ coll.}) \) decreases at \( G = 0 \) from the value \( 1 - e^{-\delta} \) with negative slope equal at least \( \frac{\delta}{4}(1 - e^{-\delta/4})e^{-2\delta} \)

- **[Peak throughput scaling compared to single BS]:**
  - \( 1 - \varepsilon \) coverage
  - Throughput \( \geq \frac{1 - \varepsilon}{\ln(1/\varepsilon)} \times m \times \) throughput of single-BS frame slotted ALOHA
Multi-Base Station Model: Decoding via Spatio-Temporal Cooperation

SA-MBS w Spatio-Temporal Cooperation
- Erasure decoding across the whole graph after each frame

Each base station is doing:
1) Temporal decoding
2) Spatial decoding
Interchangeably...
Decoding via Spatio-Temporal Cooperation

Spatio-Temporal Cooperation decoding algorithm

One iteration at arbitrary base station after each frame of $\tau$ slots

1) **Temporal SIC and Transmit**: BS $j$ performs Temporal SIC across its received slots within the frame. The set of recovered users is shared with neighboring BS’s and BS $j$ goes to next step.

2) **Check Termination**: If all the slots are recovered, BS $j$ leaves the algorithm.

3) **Receive and Spatial IC**: BS $j$ scans all the received messages from its neighbors and identifies distinct set of yet unrecovered user signals $x_u$. Then it removes all the signals from this set from all the slots where these users were active (activation slots are known for collected users) and goes to step one in the next iteration.

**Fully Distributed**: base stations communicate only with neighboring base stations!
Main results

Spatio-Temporal Cooperation:

- Users apply fixed (temporal) degree distribution $\Lambda(x)$
- [Lower Bound on $P(U_i \text{ coll.})$]:
  \[ P(U_i \text{ coll.}) \geq 1 - e^{-\delta} - P_S(H = 4\delta G) \]

- [Threshold Load]:
  \[ G^*(\delta) \geq \frac{1H^*}{4\delta} \]
  The probability $P(U_i \text{ coll.})$ stays at the maximum value $1 - e^{-\delta}$ at least in the range $[0, \frac{1H^*}{4\delta}]$

- [Peak throughput scaling compared to single BS w iterative IC]
  - $1 - \varepsilon$ coverage
  - Throughput $\geq \frac{1}{4} \frac{1-\varepsilon}{\ln(1/\varepsilon)} \times m \times$ throughput of single-BS frame slotted ALOHA with iterative interference cancellation
Optimal user degree distributions

\[ \delta = mr^2 \pi \] — average users’ spatial degree
Throughput vs Load

Simulation setup

- $m=40$, $r\approx 0.1$ (average coverage=3), $\tau=40$, $\Lambda_2=1$ (exactly two attempts per frame to send its packet)
Outline

- Single Base-Station Model
  - Slotted ALOHA w SIC
  - LDPC Codes

- Multiple Base-Station Model
  - Cooperative Slotted ALOHA
  - Codes on Random Geometric Graphs

- Ongoing/Future Work
Ongoing/Future Work

SA-MBS w Beamforming

• Instead of active users being heard in a disk of radius $r$:

Asynchronous SA-MBS

• Slot-synchronous SA-MBS: Reasonable approximation only in very dense small cell networks with low data rates

• Asynchronous SA-MBS: User packets do not arrive simultaneously at different BS
Slotted ALOHA in Small Cell Networks: How to Design Codes on Random Geometric Graphs?

Dejan Vukobratović
Associate Professor,
DEET-UNS
University of Novi Sad, Serbia

Joint work with
Dragana Bajović and Dušan Jakovetić

DLR/TUM Workshop, Munich, 26.07.2016