Random access on graphs: Capture-or tree evaluation

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Preliminaries

• $N$ users
  – Each user wants to send a packet over shared medium
  – Equal length packets
  – Users are synchronized

• Random access
  – Distributed, decentralized
  – Users behave in the same way
Framed slotted ALOHA

H. Okada, Y. Igarashi, Y. Nakanishi, ”Analysis and application of framed ALOHA channel in satellite packet switching networks”, Electronics and Communications, 1977

• Slots are organized in frames
• Each users transmits (just once) in a randomly selected slot of the frame
• Slots can be idle, singleton, or collision slots
• Collision channel model:
  – singleton slots are perfectly decoded,
  – collision slots can not be decoded at all
• Throughput:
  – No. resolved users vs no. slots:

\[ T_{max} = \frac{1}{e} \approx 0.37 \text{ (when } N = M) \]
Content resolution diversity slotted ALOHA


• Users repeat their transmission in several randomly chosen slots of the frame
  – Same number of packet replicas per user

• Collisions can be exploited!
  – Successive interference cancellation (assumed to be perfect)
  – Improves throughput
  – $T \approx 0.55$ for CRDSA with two repetitions per user
Successive interference cancellation

Analogous to iterative belief-propagation decoding of erasure correcting codes!
Irregular repetition slotted ALOHA


• Generalization of CRDSA
  – No. of replicas varies across users
  – Every user selects its no. of replicas according to a predefined distribution

• Only the distribution of the user degrees can be controlled (designed):
  – \( \Lambda_k = P[|u| = k] \)
  – \( \Lambda(x) = \sum_k \Lambda_k x^k \)

• Optimal user degree distributions are designed according to the distribution design principles used for left-irregular LDPC codes
  – Achieve asymptotic throughput close to 1
And-or tree evaluation: A tool for the asymptotic analysis

Our graphs are not trees! There are loops, i.e., there are interdependencies among messages. The results obtained by the and-or tree evaluation provide upper bound on the performance.


Probability of a packet recovery: $P_R = 1 - \lim_{i \to \infty} q(i)$, where $q(0) = 1$
And-or tree evaluation: Message update probabilities

**OR nodes**

- Degree $k$

- $q^{(k)} = p^{k-1}$

- $q = \sum_k \lambda_k p^{k-1} = \lambda(p)$

**AND nodes**

- Degree $j$

- $p^{(j)} = 1 - (1 - q)^{j-1}$

- $p = 1 - \sum_j \omega_j (1 - q)^{j-1}$
Departing from the collision channel model

\[ p = 1 - \sum_{j} \omega_j \sum_{t=0}^{j-1} \pi_{t,j} \left( \frac{j}{t} - 1 \right) q^t (1 - q)^{j-t-1} \]

- probability that user packet is decoded in a slot of degree \( j \), when \( t \) interfering packets remain and \( j - t - 1 \) are cancelled
  - i.e., capture probability

Departing from the collision channel model

No capture effect

With capture effect

inter-slot IC

capture + intra-slot IC

capture

unresolved user

resolved user
Departing from the collision channel model: Threshold-based capture effect

- Non-equal channel gains (due to fading)
- Noise is not neglected
- SIC is not perfect
- Threshold-based model of the capture-effect:
  - Packet of user \( u_i \) is captured in slot if the following condition is satisfied:

\[
\frac{P_i}{P_n + \sum_j P_j + \sum_k Q_k} \geq b
\]
Capture-Or Tree Evaluation

• There is a (fairly involved) method to derive capture probabilities in a case of:
  – Equal expected powers at the point of reception for all users,
  – Proportional residual interference power
  – Threshold-based model of the capture effect

• The method is based on the approach presented in:

• For some special cases, it can be done in a simpler way

\[
p = 1 - \sum_{j} \omega_j \sum_{t=0}^{j-1} \pi_{t,j} \binom{j-1}{t} q^t (1 - q)^{j-1-t}
\]
Capture-Or Tree Evaluation

  – Capture threshold \( b \geq 1 \) (narrowband single-antenna system)
  – Rayleigh fading scenario:
    • pdf of SNR user \( u_i \) at the reception point:
      \[
p_{X_i}(x) = \frac{1}{\gamma} e^{-\frac{x}{\gamma}}, \quad x \geq 0
\]
    • \( \gamma \) – the expected SNR
  – Perfect IC
Case study:
Frameless ALOHA


- Idea: Apply paradigm of rateless codes to slotted ALOHA:
  - No predefined frame length
  - Slots are successively added until a criterion related to performance parameters of the scheme is satisfied
  - Optimization of the slot-access probability and termination criterion
Frameless ALOHA: Optimization of the slot access probability

- The simplest case:
  - All users use the same slot access probability $p_a$ for all the slots
    - $p_a = \frac{\beta}{N}$
    - $\beta$ is the average slot degree

- Goal: Maximize throughput $T$
  
  \[ T = \frac{N_R}{M} = \frac{P_R}{M} N \]

  - $N_R$ is the number of resolved users (transmissions)
  - $P_R$ is the probability of user resolution

- Select $\beta$ such that throughput is maximized
Frameless ALOHA: Optimization of the slot access probability

- $b = 1$
- $\gamma = 5dB$
Frameless ALOHA: Optimization of the slot access probability

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<th>$b$</th>
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<td>$T_F$</td>
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Frameless ALOHA: Optimization of the slot access probability

- $b = 0.1$
  - Spread spectrum system

![Graph showing the performance of different models with varying SNR and residual interference.]
Conclusions

• There is a way to analytically assess the asymptotic performance of SIC-enabled slotted ALOHA schemes beyond the collision channel model

• Results show that, in the cases with $b \geq 1$ and low residual interference power, the scheme favors collisions

• For $b \ll 1$, there seems to be no gain to use protocol designed to exploit inter-slot IC (intra-slot IC is enough)

• Finite-length performance?
FIN