On Surrogate Channels for Code Design

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Outline

- **Popular Surrogate Channels**: BEC and biAWGN
- **Design Challenge**: Polar Codes for 8-ASK
- First Attempt
- Second Attempt
- **Detour**: Mismatched Decoding
- Final Attempt
- Conclusions
BEC and biAWGN

- **Idea:** analyze decoding by tracking mutual information (MI) as it propagates through factor graph.
- **Two channels are “easy”:**
  - Binary Erasure Channel (BEC)
  - biAWGN Channel (with Gaussian Approximation)\(^1\)

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Observation: codes often have “universal” properties.\textsuperscript{3}

Idea: Design code for surrogate channel, use it for target channel.\textsuperscript{4}

Higher-order modulation: use surrogate channel for each bit-channel.\textsuperscript{5}


\textsuperscript{5} F. Steiner, G. Böcherer, and G. Liva, “Protograph-based LDPC code design for shaped bit-metric decoding,” 2016.
Design Challenge: Polar Code for 8-ASK\(^6\)

For the following modulation scheme, we want to **efficiently** construct Polar codes that perform as good as polar codes constructed by exhaustive search ("Monte Carlo Construction").

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1.5 bpcu, 8-ASK, \( c = 1/2 \), \( n = 512 \)
First Attempt

- Three bit-channels $p_{Y|B_i}, i = 1, 2, 3$.
- Replace the $i$th bit-channel by a biAWGN surrogate channel with MI equal to $I(B_i; Y), i = 1, 2, 3$. 
1.5 bpcu, 8-ASK, $c = 1/2$, $n = 512$

Monte Carlo Construction
bit-channel surrogate (MI)
Second Attempt

- Replace the polarized bit-channels by biAWGN surrogate channels with MI equal to

\[ I(\tilde{B}_1; Y) \]
\[ I(\tilde{B}_2; Y|\tilde{B}_1) \]
\[ I(\tilde{B}_3; Y|\tilde{B}_1 \tilde{B}_2) \]

respectively.
1.5 bpcu, 8-ASK, $c = 1/2$, $n = 512$

![Graph showing FER versus $E_s/N_0$ for different methods: Monte Carlo construction, bit-channel surrogate (MI), and polarized [...].]
Detour: Mismatched Decoding

What is mutual information?

- Memoryless channel $p_{Y|X}$.
- Random codebook $\mathcal{C} = \{X^n(1), \ldots, X^n(2^{nR})\}$ with entries iid $P_X$.
- Message $w \in \{1, 2, \ldots, 2^{nR}\}$
- ML decoder

$$
\hat{W} = \arg\max_{w \in \{1, 2, \ldots, 2^{nR}\}} p_{Y^n|X^n}(Y^n|X^n(w)) = \prod_{i=1}^{n} p_{Y|X}(Y_i|X_i(w))
$$

- Error probability $\Pr(W \neq \hat{W}) \xrightarrow{n \to \infty} 0$ if $R < I(X; Y)$. 
Detour: Mismatched Decoding\textsuperscript{7,8}

- (Random coding as on previous slide).
- Auxiliary channel $q(\cdot | \cdot)$
- Mismatched Decoder

\[
\hat{W} = \arg\max_{w \in \{1,2,\ldots,2^{nR}\}} \prod_{i=1}^{n} q_{Y|X}(Y_i|X_i(w))
\]

- Achievable rate

\[
R_{LM} = \max_{s,r} E \left[ \log \frac{q(Y|X)^s r(X)}{q_{s,r}(Y)} \right]
\]

- Auxiliary output distribution $q_{s,r}(\cdot) = E[q(\cdot | X)^s r(X)]$
- $s \geq 0$, function $r : \mathcal{X} \to \mathbb{R}$.

Detour: Mismatched Decoding

- Let’s account for what the decoder is actually doing!
- $L$-value defines auxiliary channel via

$$L = \log \frac{q_{B|Y}(0|y)}{q_{B|Y}(1|y)}$$

$$q_{B|Y}(b|y) = \frac{e^{-L \cdot b}}{1 + e^{-L}}$$

$\Rightarrow$ Estimate $R_{LM}$ for $L$-value as decoding metric.
Final Attempt

- Replace each polarized bit-channel by a biAWGN surrogate channel with MI matched to $R_{LM}$ estimated from $\hat{L}_i$, $i = 1, 2, 3$. 
1.5 bpcu, 8-ASK, $c = 1/2$, $n = 512$

FER

Monte Carlo construction
bit-channel surrogate (MI)
polarized [...] (MI)
polarized [...] (R_{LM})

$E_s/N_0$ [dB]
Conclusions

- Monte Carlo construction: half an hour (implementation in C).
- Surrogate channel design: some milliseconds.
- Construction with appropriate surrogate channel yields performance as good as Monte Carlo construction.
- Tool: information theory for mismatched decoding.
Literature


