Decoder-in-the-Loop: Genetic Optimization-based Code Design

A. Elkelesh, M. Ebada, S. Cammerer and Stephan ten Brink
Institute of Telecommunications
University of Stuttgart

OWHTC 2019, DLR, Oberpfaffenhofen

28.02.2019
Outline

1 Introduction

2 Genetic Algorithm-based Polar Code Construction

3 Results for different decoders and channels

4 Decoding Complexity Reduction

5 Genetic Algorithm-based LDPC Code Design

6 Summary
Agenda

1 Introduction

2 Genetic Algorithm-based Polar Code Construction

3 Results for different decoders and channels

4 Decoding Complexity Reduction

5 Genetic Algorithm-based LDPC Code Design

6 Summary
Polar Codes

- Polar codes were introduced by Arıkan.
- Asymptotically achieve capacity under SC decoding.
- Low encoding and SC decoding complexity $\mathcal{O}(N \log N)$.
- Based on the concept of channel polarization.
  - Uncoded information bits are transmitted over the reliable (noiseless) bit channels.
  - Frozen (known) bits are transmitted over the unreliable (noisy) bit channels.

http://webdemo.inue.uni-stuttgart.de/webdemos/08_research/polar/
Polar Decoding

- SC decoder: achieves capacity for infinite length codes.
- Belief propagation (BP) decoder: better BER performance than SC for finite length codes.
- Successive cancellation list (SCL) decoder: approaches the ML decoder performance.
- SCL decoding of the modified polar code (outer high rate CRC code concatenated with inner polar code): outperforms the ML decoder of pure polar codes.
Polar Code Construction

- Finding the best $k$ bit positions for information transmission.
  - The remaining $N - k$ bit positions are frozen.
- State-of-the-art design methods assume SC decoding.
  - Thus, not necessarily optimal under BP and SCL decoding.
  - Decoder-tailored code design will enhance the performance!
Frozen/non-frozen set “Frozen Channel Chart”

Bhattacharyya-based design $A = [0 0 0 1 0 1 1 1]$ (i.e., $R_c = 0.5$)

Random design $A = [1 0 1 0 1 0 1 0]$ (i.e., $R_c = 0.5$)

- For the $\mathcal{D}(8,4)$-code, the information set $A = \{4, 6, 7, 8\}$
  - can be represented as $A = [0 0 0 1 0 1 1 1]$
- Code rate $R_c = \frac{\sum A}{N}$
- Bit-channels are sorted with decreasing Bhattacharyya values. Colored: frozen “0”; white: non-frozen “1”
Agenda

1. Introduction

2. Genetic Algorithm-based Polar Code Construction

3. Results for different decoders and channels

4. Decoding Complexity Reduction

5. Genetic Algorithm-based LDPC Code Design

6. Summary
Abstract view

- **Parameters for optimization**
- **Initial population, e.g.**:
  - Bhattacharyya-based design
  - RM-Polar codes

\[
\text{BERs} = \text{ComputeBERs}(P_i, SNR_{GenAlg})
\]

![Diagram showing steps of the optimization process](image)
Evolutionary Transformations

(a) Mutation or swapping  
(b) Crossover

- Code rate maintained (i.e., stays fixed)
Agenda

1. Introduction
2. Genetic Algorithm-based Polar Code Construction
3. Results for different decoders and channels
4. Decoding Complexity Reduction
5. Genetic Algorithm-based LDPC Code Design
6. Summary
BP-tailed Polar Codes

- GenAlg-based construction of a $\mathcal{P}(2048,1024)$-code under BP ($N_{\text{it},\max} = 200$) decoding over the AWGN channel and no CRC is used.
Epochs of Genetic Optimization

- Evolution of the BER at $SNR_{GenAlg} \left( \frac{E_b}{N_0} \right) = 2\, \text{dB}$.
- each code candidate was simulated to count at least 1000 bit errors.
**SCL-tailored Polar Codes**

<table>
<thead>
<tr>
<th>Decoder</th>
<th>Construction @ design SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCL ($L = 32$)</td>
<td>Tal and Vardy @ 2 dB</td>
</tr>
<tr>
<td>SCL ($L = 32$)</td>
<td>RM-Polar @ 2 dB</td>
</tr>
<tr>
<td>SCL ($L = 32$)</td>
<td>GenAlg @ 2 dB [No CRC needed!]</td>
</tr>
<tr>
<td>SCL+CRC-16 ($L = 32$)</td>
<td>Tal and Vardy @ 2 dB</td>
</tr>
</tbody>
</table>

- BER performance of the GenAlg-based $\mathcal{P}(2048,1024)$-code under SCL decoding over the AWGN channel.
- Note that the CRC-aided polar code ($- * -$): $N = 2048$, $k = 1024$, $r = 16$, $R_c = 0.5$ and, thus, the polar code is a $\mathcal{P}(2048,1040)$-code.
## Decoder-tailored Polar Codes

<table>
<thead>
<tr>
<th>Construction @ design SNR</th>
<th>Decoder to reach BER $10^{-4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SC</td>
</tr>
<tr>
<td>Bhattacharyya @ 3.6 dB</td>
<td>2.7 dB</td>
</tr>
<tr>
<td>Tal and Vardy @ 2 dB</td>
<td>2.65 dB</td>
</tr>
<tr>
<td>GenAlg BP-tailored @ 2 dB</td>
<td>&gt; 9 dB</td>
</tr>
<tr>
<td>GenAlg SCL-tailored @ 2 dB</td>
<td>&gt; 6 dB</td>
</tr>
</tbody>
</table>

- Illustration of polar design and decoder architecture mismatch by evaluating the minimum $E_b/N_0$ required to achieve a target BER of $10^{-4}$ for a $\mathcal{P}(2048,1024)$-code over AWGN channel.
Frozen Channel Chart

- Frozen bit position pattern of a $\mathcal{P}(2048,1024)$-code with different polar code construction algorithms. The 2048 bit positions are plotted over a $16 \times 128$ matrix. Note that the bit-channels are sorted with decreasing Bhattacharyya parameter value. Colored: frozen; White: non-frozen.
Weight enumerators

<table>
<thead>
<tr>
<th>Construction @ design SNR</th>
<th>$d_{\text{min}}$</th>
<th>$A_8$</th>
<th>$A_{16}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tal and Vardy @ 2 dB</td>
<td>16</td>
<td>0</td>
<td>11648</td>
</tr>
<tr>
<td>GenAlg BP-tailored @ 2 dB</td>
<td>8</td>
<td>8</td>
<td>773</td>
</tr>
<tr>
<td>GenAlg SCL-tailored @ 2 dB</td>
<td>16</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

- Using the algorithm described in [2]
- GenAlg reduces number of low-weight codewords!

BP-tailored Polar Codes

- BLER performance of the GenAlg-based $\mathcal{P}(1024,512)$-code under BP decoding over the Rayleigh fading channel and no CRC is used.
SCL-tailed Polar Codes

<table>
<thead>
<tr>
<th>Decoder</th>
<th>Construction @ design SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sequential (L = 32)</td>
<td>Gaussian Approximation @ 5 dB</td>
</tr>
<tr>
<td>SCL (L = 32)</td>
<td>Bhattacharyya @ 3.6 dB</td>
</tr>
<tr>
<td>SCL (L = 32)</td>
<td>$\beta$-expansion @ $\beta = 2^{0.25}$</td>
</tr>
<tr>
<td>SCL (L = 32)</td>
<td>5G specification of 3GPP</td>
</tr>
<tr>
<td>SCL (L = 32)</td>
<td>GenAlg @ 5 dB [No CRC needed!]</td>
</tr>
<tr>
<td>SCL+CRC-24 (L = 32)</td>
<td>5G specification of 3GPP</td>
</tr>
</tbody>
</table>

- BLER performance of the GenAlg-based $P(1024,512)$-code under SCL decoding over the Rayleigh fading channel.
- Note that the CRC-aided polar code (- - - ): $N = 1024$, $k = 512$, $r = 24$, $R_c = 0.5$ and, thus, the polar code is a $P(1024,536)$-code.
1. Introduction
2. Genetic Algorithm-based Polar Code Construction
3. Results for different decoders and channels
4. Decoding Complexity Reduction
5. Genetic Algorithm-based LDPC Code Design
6. Summary
BP Decoding

- BER performance of the GenAlg-based $\mathcal{P}(2048,1024)$-code under BP decoding with reduced $N_{it,max}$ over the AWGN channel and no CRC is used.
**SCL Decoding**

<table>
<thead>
<tr>
<th>Decoder</th>
<th>Construction @ design SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCL ($L = 32$)</td>
<td>Tal and Vardy @ 2 dB</td>
</tr>
<tr>
<td>SCL ($L = 4$)</td>
<td>Tal and Vardy @ 2 dB</td>
</tr>
<tr>
<td>SCL ($L = 4$)</td>
<td>GenAlg @ 2 dB</td>
</tr>
</tbody>
</table>

- BER performance of the GenAlg-based $P(2048,1024)$-code under SCL decoding with reduced list size $L$ over the AWGN channel and no CRC is used.
For more details

Agenda

1. Introduction
2. Genetic Algorithm-based Polar Code Construction
3. Results for different decoders and channels
4. Decoding Complexity Reduction
5. Genetic Algorithm-based LDPC Code Design
6. Summary
LDPC code design

- We design the whole parity-check matrix (i.e., $H$-matrix)
  - No degree profile optimization (e.g., EXIT charts)
  - No PEG algorithm used
**Results** \((N = 128, R_c = 0.5)\)

<table>
<thead>
<tr>
<th>Decoder</th>
<th>Construction @ design SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>(BP \ (N_{it,max} = 200))</td>
<td>(3,6) Regular LDPC Code [1]</td>
</tr>
<tr>
<td>(BP \ (N_{it,max} = 200))</td>
<td>CCSDS Up-Link LDPC Code [1]</td>
</tr>
<tr>
<td>(BP \ (N_{it,max} = 200))</td>
<td>AR3A LDPC Code [1]</td>
</tr>
<tr>
<td>(BP \ (N_{it,max} = 200))</td>
<td>ARJA LDPC Code [1]</td>
</tr>
<tr>
<td>(BP \ (N_{it,max} = 200))</td>
<td>GenAlg Irreg LDPC @ 3 dB</td>
</tr>
</tbody>
</table>

- No special graph structure

Results \( (N = 128, R_c = 0.5) \)

<table>
<thead>
<tr>
<th>Decoder</th>
<th>Construction @ design SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>BP ( (N_{it,\text{max}} = 200) )</td>
<td>(3,6) Regular LDPC Code [1]</td>
</tr>
<tr>
<td>BP ( (N_{it,\text{max}} = 200) )</td>
<td>CCSDS Up-Link LDPC Code [1]</td>
</tr>
<tr>
<td>BP ( (N_{it,\text{max}} = 200) )</td>
<td>AR3A LDPC Code [1]</td>
</tr>
<tr>
<td>BP ( (N_{it,\text{max}} = 200) )</td>
<td>ARJA LDPC Code [1]</td>
</tr>
<tr>
<td>BP ( (N_{it,\text{max}} = 20) )</td>
<td>GenAlg Irreg LDPC @ 3 dB</td>
</tr>
</tbody>
</table>

- No special graph structure

Results \((N = 128, R_c = 0.5)\)

- No special graph structure

Results \((N = 128, R_c = 0.5)\)

<table>
<thead>
<tr>
<th>Decoder</th>
<th>Construction @ design SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>BP ((N_{it,\text{max}} = 200))</td>
<td>(3,6) Regular LDPC Code [1]</td>
</tr>
<tr>
<td>BP ((N_{it,\text{max}} = 200))</td>
<td>CCSDS Up-Link LDPC Code [1]</td>
</tr>
<tr>
<td>BP ((N_{it,\text{max}} = 200))</td>
<td>AR3A LDPC Code [1]</td>
</tr>
<tr>
<td>BP ((N_{it,\text{max}} = 200))</td>
<td>ARJA LDPC Code [1]</td>
</tr>
<tr>
<td>BP ((N_{it,\text{max}} = 200))</td>
<td>GenAlg RA code @ 3 dB</td>
</tr>
</tbody>
</table>

- RA graph structure
  - Similar to LDPC codes from DVB-S.2 standard

For more details


- https://github.com/AhmedElkelesh/Link-will-be-Available-After-Review
Agenda

1. Introduction
2. Genetic Algorithm-based Polar Code Construction
3. Results for different decoders and channels
4. Decoding Complexity Reduction
5. Genetic Algorithm-based LDPC Code Design
6. Summary
Summary

- New polar code construction algorithm
  - the resulting codes are decoder-tailored and channel-tailored
  - BP-tailored Polar Codes
  - SCL-tailored Polar Codes
  - outperforms the state-of-the-art construction algorithms

- Codes can be designed with the aim of reducing the decoding complexity

- Can be used to design LDPC codes
  - designing the $H$-matrix
  - no EXIT curves matching
  - no PEG used
Thank you for your attention!
Backup Slide (Reference polar codes)

References