One and Two Bit Message Passing Decoding for SC-LDPC Codes and Higher-Order Modulation

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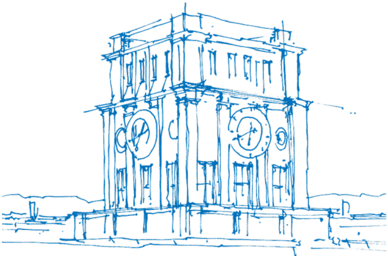
OWHTC February 28th, 2019



Introduction

- High throughput applications (e.g., optical communications) require low-complexity decoders.

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- Data flow within an LDPC decoder[1]:

$$F_{\mathsf{LDPC}} = \frac{2 \cdot n_{\mathsf{c}} \cdot D \cdot q \cdot d_{\mathsf{avg}}}{R_{\mathsf{c}}}$$

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- Well, but why shouldn't we still exploit some of the soft-information?

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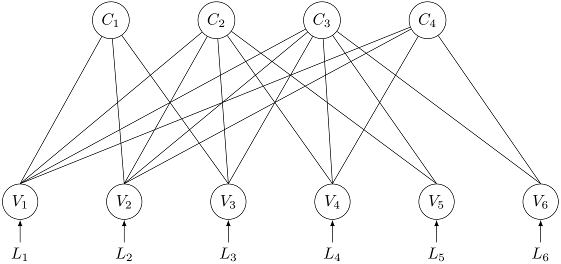
One and Two Bit Message Passing

Binary Message Passing (BMP) (I)[2]



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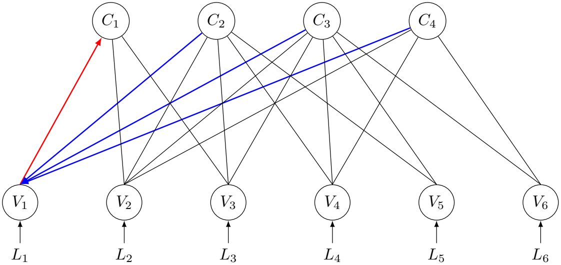
Binary Message Passing (BMP) (I)[2]

Variable node update



$$m^{(\ell)}_{V_i \to C_j} = \begin{cases} +1, & \text{if } \left(\sum_{j' \in \mathcal{N}(C_j)} D^{(\ell)} \cdot m^{(\ell)}_{C_{j'} \to V_i} + L_i\right) > 0 \\ -1, & \text{if } \left(\sum_{j' \in \mathcal{N}(C_j)} D^{(\ell)} \cdot m^{(\ell)}_{C_{j'} \to V_i} + L_i\right) < 0 \end{cases}$$

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Binary Message Passing (BMP) (I)[2]

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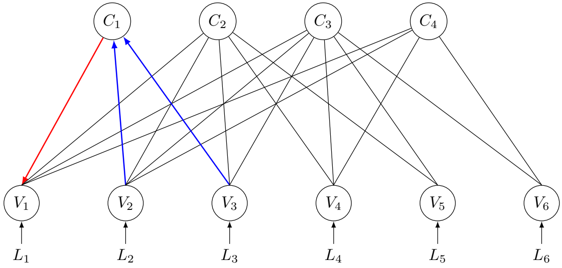

$$m^{(\ell)}_{C_j \to V_i} = \prod_{i' \in \mathcal{N}(C_j)} m^{(\ell)}_{V_{i'} \to C_j} \qquad \boxed{\in \{-1, +1\}}$$

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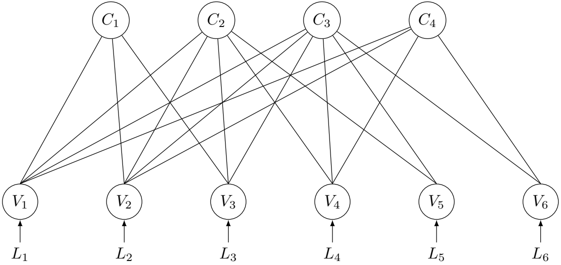
- Weighting factor $D^{(\ell)}$ for each iteration is calculated off-line via density evolution and stored.

Ternary Message Passing (TMP) (I)



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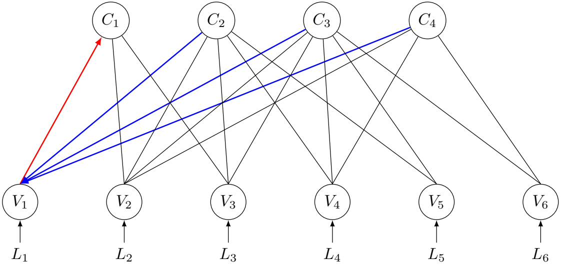
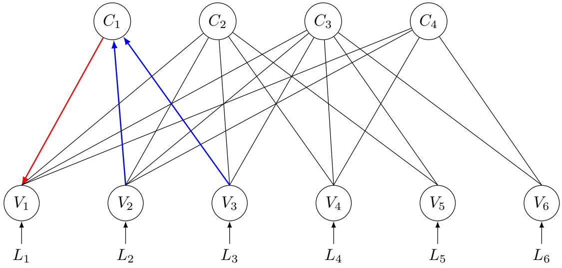


$$m_{V_i \to C_j}^{(\ell)} = \begin{cases} +1, & \text{if } \left(\sum_{j' \in \mathcal{N}(V_i) \setminus \{j\}} D^{(l)} \cdot m_{C_{j'} \to V_i}^{(\ell)} + L_i \right) > a \\ -1, & \text{if } \left(\sum_{j' \in \mathcal{N}(V_i) \setminus \{j\}} D^{(l)} \cdot m_{C_{j'} \to V_i}^{(\ell)} + L_i \right) < -a \\ 0, & \text{else.} \end{cases}$$

Ternary Message Passing (TMP) (I)

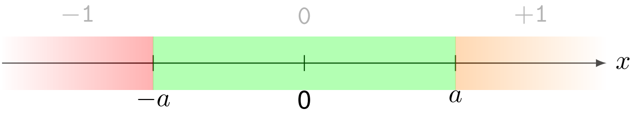
Check node update (same as BMP)



$$m^{(\ell)}_{C_j \to V_i} = \prod_{i' \in \mathcal{N}(C_j) \setminus \{i\}} m_{V_{i'} \to C_j}$$

Ternary Message Passing (TMP) (II)

- Region between $-a$ and a is declared to be an erasure region (complete uncertainty).



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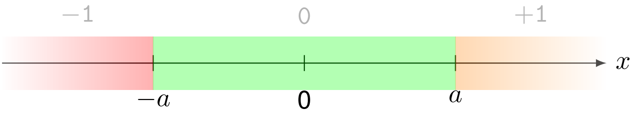
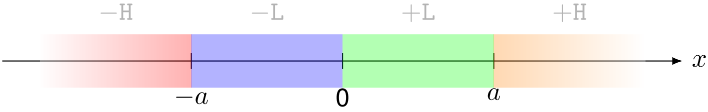


- Extrinsic channel is a binary error and erasure channel (BEEC).
- Straightforward to use with punctured LDPC codes or for channels with erasures (strong fading).

Quaternary Message Passing (QMP) (I)

- TMP already requires two bits. Why not introduce another level?
- We devide the real line into four regions:



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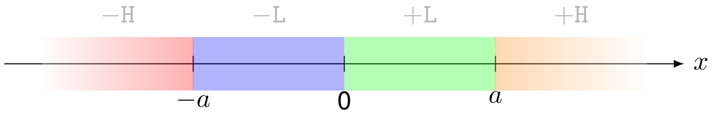
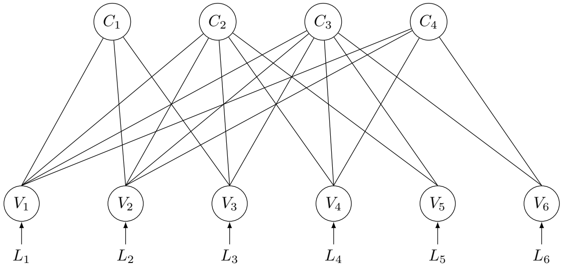


- Extrinsic channel is a symmetric quaternary output channel.

Quaternary Message Passing (QMP) (II)



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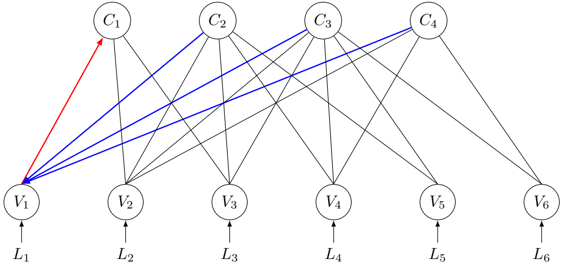
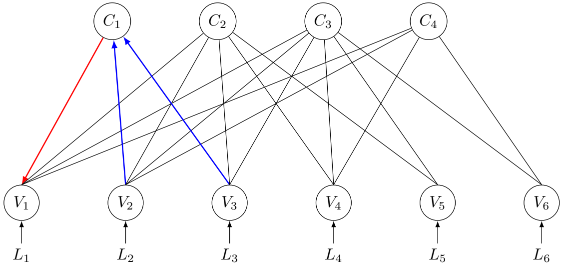


$$m_{V_i \to C_j}^{(\ell)} = \begin{cases} -\mathtt{H}, & \left(\sum_{j' \in \mathcal{N}(V_i) \setminus \{j\}} D^{(l)} \cdot m_{C_{j'} \to V_i}^{(\ell)} + L_i\right) \leq -a \\ -\mathtt{L}, & -a < \left(\sum_{j' \in \mathcal{N}(V_i) \setminus \{j\}} D^{(l)} \cdot m_{C_{j'} \to V_i}^{(\ell)} + L_i\right) < 0 \\ +\mathtt{L}, & 0 \leq \left(\sum_{j' \in \mathcal{N}(V_i) \setminus \{j\}} D^{(l)} \cdot m_{C_{j'} \to V_i}^{(\ell)} + L_i\right) < a \\ +\mathtt{H}, & \left(\sum_{j' \in \mathcal{N}(V_i) \setminus \{j\}} D^{(l)} \cdot m_{C_{j'} \to V_i}^{(\ell)} + L_i\right) \geq a \end{cases}$$

Quaternary Message Passing (QMP) (II)

Check node update (classical min-sum)



$$m^{(\ell)}_{C_j \to V_i} = \min_{V_{i'} \in \mathcal{N}(C_j) \setminus \{i\}} \left| m^{(\ell-1)}_{V_{i'} \to C_j} \right| \times \prod_{V_{i'} \in \mathcal{N}(C_j) \setminus \{i\}} \operatorname{sign}\left(m^{(\ell-1)}_{V_{i'} \to C_j} \right)$$

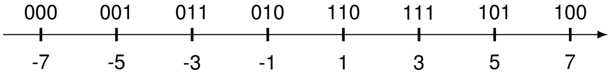
Higher Order Modulation

System Model

- Additive white Gaussian noise channel with N_i iid. $\mathcal{N}(0, \sigma^2)$.

$$Y_i = X_i + N_i, \qquad i = 1, \ldots, n.$$

- Discrete signaling with constellation $\mathcal{X}$: $M = 2^m$-ASK.



- Binary reflected Gray code (BRGC) labeling.
- Mapping of constellation point $x \in \mathcal{X}$ to its label via

$$\boldsymbol{b} = (b_1 b_2 \ldots b_m) = \chi(x) \in \{0, 1\}^m.$$

Bit-Metric Decoding

- Bit-metric, soft-decision (SD) decoders:

$$l_j(y) = \log \frac{P_{B_j|Y}(0|y)}{P_{B_j|Y}(1|y)}, \quad j = 1, \ldots, m.$$

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- The reliability of each bit-level is different.
- The PDF of the soft-information is not symmetric, i.e.,

$$p_{L|B}(l|0) \neq p_{L|B}(-l|1).$$

Structured Ensembles: Protographs

- Structured LDPC codes (e.g., protograph-based, multi-edge type) are particularly suited for the optimization with different reliabilities.

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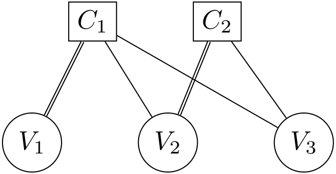

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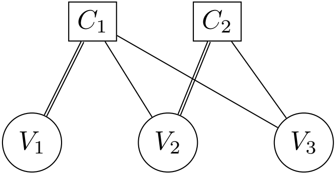

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- Final parity-check matrix derived via (cyclic) lifting operation.

Density Evolution for BMP/TMP/QMP (I)

- Density evolution is simplified when the operations at VNs and CNs and the soft-information is symmetric.

[3] J. Hou, P. H. Siegel, L. B. Milstein, and H. D. Pfister, “Capacity-approaching bandwidth-efficient coded modulation schemes based on low-density parity-check codes,” IEEE TCOM, vol. 49, no. 9, pp. 2141–2155, Sep. 2003.

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- Use channel adapters:[3]

$$\tilde{L}_j = L_j \cdot (1 - 2B_j).$$

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Track the evolution of the following probabilities for all edges $[\boldsymbol{B}]_{ij} \neq 0, i \in \{1, \ldots, m_{\mathsf{P}}\}, j \in \{1, \ldots, n_{\mathsf{P}}\}$.

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Input Parameters for DE (I)

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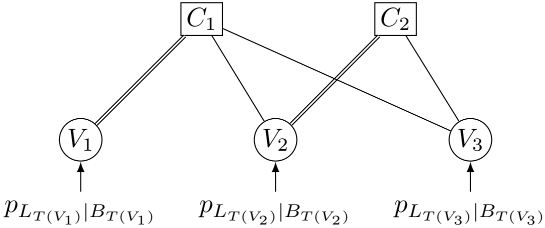
- Example: 8-ASK, $m = 3$ bit channels.



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$$= \sum_z \Pr\left\{L_{\text{in}}^{(\ell)} = z\right\} \int_{-a-z}^{a-z} p_{\tilde{L}_{T(j)}}(\ell)\,\mathrm{d}\ell$$

$$p_{-1}^{(\ell)}(i,j) = \Pr\left\{L_{\text{ch}} + L_{\text{in}}^{(\ell)} < -a\right\}$$

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- Full description of DE for BMP/TMP/QMP in papers[4][5].

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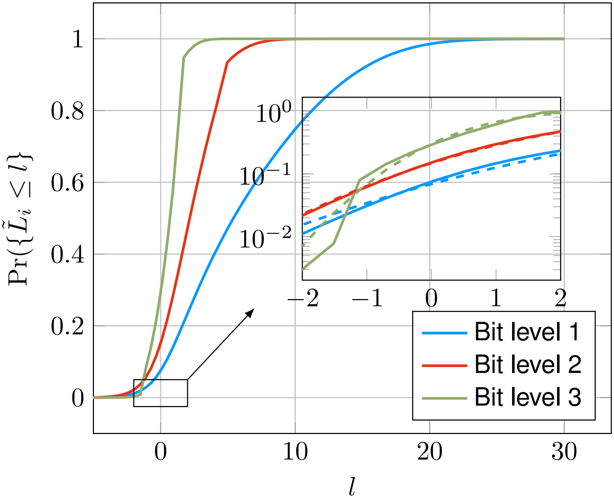
   Exemplary calculation for TMP erasure region ($\mu_{\text{ch},j} = 2/\tilde{\sigma}_j^2$ and $\sigma_{\text{ch},j}^2 = 4/\tilde{\sigma}_j^2$):

$$p_0^{(0)}(i,j) = Q\left(\frac{-a + \mu_{\text{ch},T(j)}}{\sigma_{\text{ch},T(j)}}\right) - Q\left(\frac{a + \mu_{\text{ch},T(j)}}{\sigma_{\text{ch},T(j)}}\right).$$

CDF Plot: 4-ASK uniform



Numerical Results

Setup for the Numerical Examples

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- Code class: SC-LDPC codes with VN degree four.
  - Based on protographs: [4 4 4], [4 4 4 4], [4 4 4 4 4 4].
  - Constructed via edge spreading, therefore memory 3.
  - Thresholds: Right unterminated and window decoding with $W = 15$.
  - Finite length: Terminated after $L = 50$ spatial positions.

Numerical Results: Asymptotics

Achievable rates (Unconstrained Shannon limit: 8.45 dB):

Mode	$R^{-1}_{\mathsf{BMD}}(1.5; P_X)$ [dB]	
	SD	HD
4U-0.75	9.3084	11.0325
8PS-0.67	8.5334	10.8367
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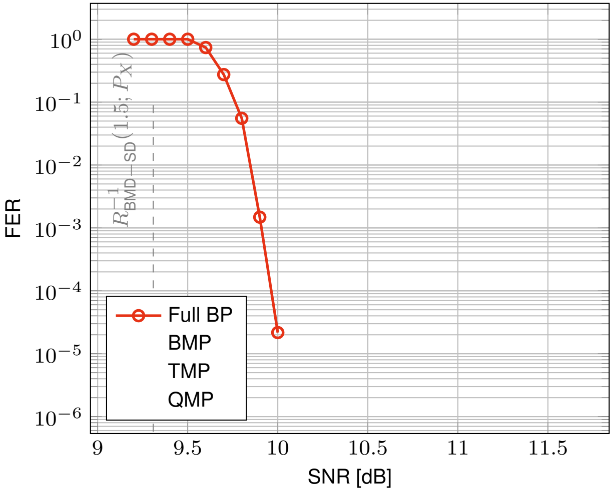
Decoding thresholds:

Mode	$\text{SNR}_{\text{th}}^{\text{full}}$	$\text{SNR}_{\text{th}}^{\text{BMP}}$	$\text{SNR}_{\text{th}}^{\text{TMP}}$	$\text{SNR}_{\text{th}}^{\text{QMP}}$
4U-0.75	9.41	10.89	10.11	10.00
8PS-0.67	8.65	10.81	9.68	9.50
8PS-0.83	8.67	10.06	9.33	9.23

Numerical Results: Finite Length 4-ASK uniform



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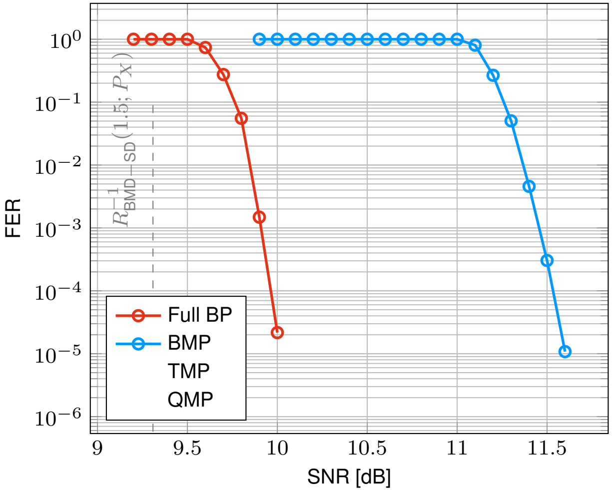
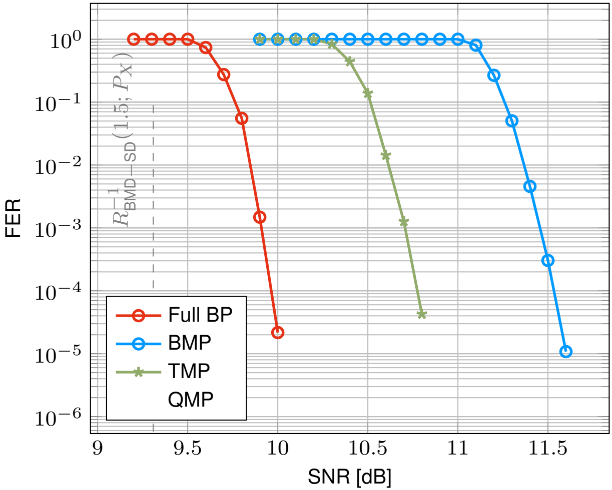


Numerical Results: Finite Length 4-ASK uniform



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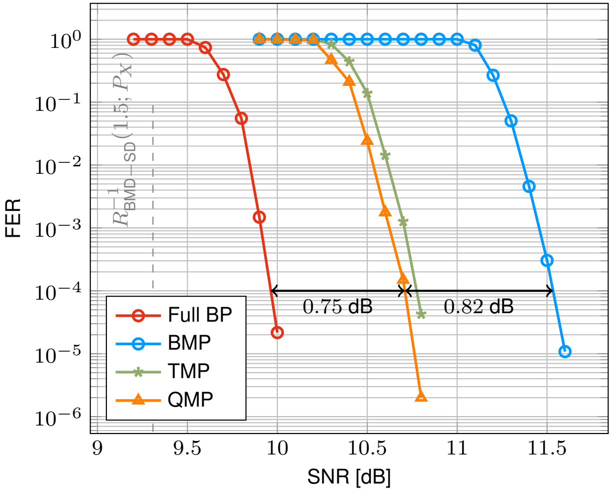
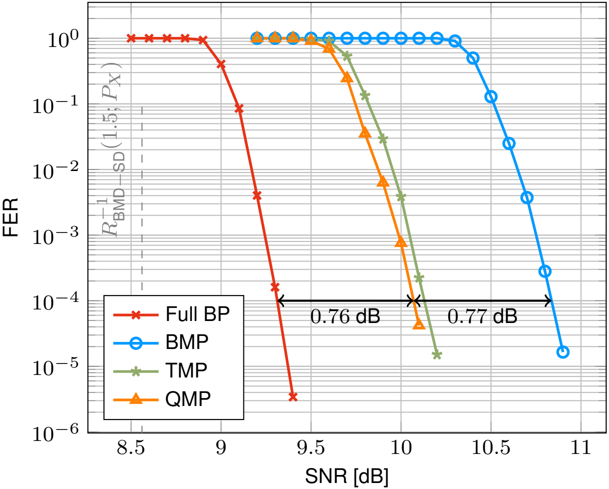


Numerical Results: Finite Length 8-ASK PS



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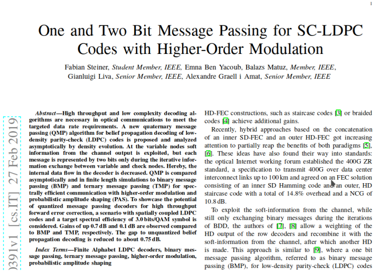

One and Two Bit Message Passing for SC-LDPC Codes with Higher-Order Modulation

Fabian Steiner, *Student Member, IEEE,* Emna Ben Yacoub, Balazs Matuz, *Member, IEEE,* Gianluigi Liva, *Senior Member, IEEE,* Alexandre Graell i Amat, *Senior Member, IEEE*

Abstract—High throughput and low complexity decoding algorithms are necessary in optical communications to meet the targeted data rate requirements. A new quaternary message passing (QMP) algorithm for belief propagation decoding of low-density parity-check (LDPC) codes is proposed and analyzed asymptotically by density evolution. At the variable nodes soft information from the channel output is exploited, but each message is represented by two bits only during the iterative information exchange between variable and check nodes. Hereby, the internal data flow in the decoder is decreased. QMP is compared asymptotically and in finite length simulations to binary message passing (BMP) and ternary message passing (TMP) for spectrally efficient communication with higher-order modulation and probabilistic amplitude shaping (PAS). To showcase the potential of quantized message passing decoders for high throughput forward error correction, a scenario with spatially coupled LDPC codes and a target spectral efficiency of 3.0 bits/QAM symbol is considered. Gains of up to 0.7 dB and 0.1 dB are observed compared to BMP and TMP, respectively. The gap to unquantized belief propagation decoding is reduced to about 0.75 dB.

Index Terms—Finite Alphabet LDPC decoders, binary message passing, ternary message passing, higher-order modulation, probabilistic amplitude shaping

HD-FEC constructions, such as staircase codes [3] or braided codes [4] achieve additional gains.

Recently, hybrid approaches based on the concatenation of an inner SD-FEC and an outer HD-FEC got increasing attention to partially reap the benefits of both paradigms [5], [6]. These ideas have also found their way into standards: the optical Internet working forum established the 400G ZR standard, a specification to transmit 400G over data center interconnect links up to 100 km and agreed on an FEC solution consisting of an inner SD Hamming code and an outer, HD staircase code with a total of 14.8% overhead and a NCG of 10.8 dB.

To exploit the soft-information from the channel, while still only exchanging binary messages during the iterations of BDD, the authors of [7], [8] allow a weighting of the HD output of the row decoders and recombine it with the soft-information from the channel, after which another HD is made. This approach is similar to [9], where a one bit message passing algorithm, referred to as binary message passing (BMP), for low-density parity-check (LDPC) codes

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