Spatial Coupling - Essential Technology for High Throughput Coding?

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Outline

1. Spatially coupled (SC) LDPC Codes

2. Non-uniformly coupled SC-LDPC codes

3. Problems with windowed decoding of SC-LDPC codes (and first solutions)

4. Conclusions and outlook
Spatially Coupled LDPC Code Ensemble

Start with LDPC Code

- We start with a regular LDPC code
  - Variable node (code bits) degree $d_v$
  - Check node (constraints) degree $d_c$
- Total number of $M$ variable nodes (code bits)
**Spatially Coupled LDPC Code Ensemble**

**Start with LDPC Code**

- We start with a regular LDPC code
  - Variable node (code bits) degree $d_v$
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- Total number of $M$ variable nodes (code bits)

- **Spatially coupled code**: replicate $L$ copies of this code along a new, spatial dimension

- $L$ denotes the *replication factor* of the code
Spatially Coupled LDPC Code Ensemble

$L$ Disjoint LDPC Codes

Spatially Coupled LDPC Code Ensemble

$L$ Disjoint LDPC Codes

Spatial coupling: connect uniformly at random each edge from variable node at SP $z$ to check node at position $\{z, z+1, \ldots, z+w-1\}$

$w$: coupling factor

Spatially Coupled LDPC Code Ensemble

Spatially Coupled LDPC Code with $w = 2$

Spatial position $z - 1$

Spatial position $z$

Spatial position $z + 1$

Spatially Coupled LDPC Code Ensemble

Terminated Spatially Coupled LDPC Code with $w = 2$ and $L = 3$

- Two extra check nodes lead to rate loss (negligible if $L$ large enough)
- Check nodes at boundary have lower degree, hence better correction capabilities

Spatially Coupled LDPC Codes are Capacity-Achieving

- Under some conditions, SC-LDPC codes are capacity-achieving [KRU11], in particular, for the decoding threshold on the binary erasure channel (BEC),

  \[
  \lim_{w \to \infty} \lim_{L \to \infty} \varepsilon_{\text{BP}}(d_v, d_c, L, W) = \lim_{w \to \infty} \lim_{L \to \infty} \varepsilon_{\text{MAP}}(d_v, d_c, L, W) = \varepsilon_{\text{MAP, uncoupl.}}(d_v, d_c)
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• Rate of the SC-LDPC code ensemble:

\[
R = \left(1 - \frac{d_v}{d_c}\right) - O(w/L)
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- Rate of the SC-LDPC code ensemble: \( R = \left( 1 - \frac{d_v}{d_c} \right) - O(w/L) \)

**Practical code constructions:**

- Keep \( L \) small, as large \( L \) can worsen finite length performance [OU15]
- For small, fixed \( L \), keep \( w \) small to keep rate loss and decoder complexity small
- Performance for small \( w \) not necessarily good
- Modified, generalized ensemble for small \( w \) required

Density Evolution for $\varepsilon = 0.48, \ L = 50$

Conventional Spatially Coupled LDPC Code
$d_v = 5, \ d_c = 10$

$I = 200$ iter.
Density Evolution for $\varepsilon = 0.48$, $L = 50$

Conventional Spatially Coupled LDPC Code
$d_v = 5, d_c = 10$

$I = 400$ iter.
Density Evolution for $\varepsilon = 0.48, \ L = 50$

Conventional Spatially Coupled LDPC Code
$d_v = 5, \ d_c = 10$

$I = 600$ iter.
Density Evolution for $\epsilon = 0.48$, $L = 50$

Conventional Spatially Coupled LDPC Code
$d_v = 5$, $d_c = 10$

$I = 800$ iter.
Density Evolution for $\varepsilon = 0.48$, $L = 50$

Conventional Spatially Coupled LDPC Code
$d_v = 5$, $d_c = 10$

$I = 1000$ iter.
Density Evolution for $\varepsilon = 0.48$, $L = 50$

Conventional Spatially Coupled LDPC Code $d_v = 5$, $d_c = 10$

$I = 1500$ iter.
Density Evolution for $\varepsilon = 0.48$, $L = 50$

Conventional Spatially Coupled LDPC Code

$d_v = 5$, $d_c = 10$

$I = 1740$ iter.
Windowed Decoding of Spatially Coupled LDPC Codes

Conventional Spatially Coupled LDPC Code
\( d_v = 5, \; d_c = 10 \)

\( I = 200 \) iter.

- Windowed decoding sufficient to achieve capacity [ISU+13]
- To save latency, we are only interested in left-most portion of wave and use windowed decoder of size \( W_D \) for this part (decode while receive)
- Window latency of order \( W_D + w (W_D + w - 1) \) SPs in window
- Decoding complexity of order \( (W_D + w) \cdot I \) (\( I \): number iterations per window)

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Windowed Decoding of Spatially Coupled LDPC Codes

Conventional Spatially Coupled LDPC Code
\( d_v = 5, d_c = 10 \)

\( I = 800 \) iter.

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- To save latency, we are only interested in left-most portion of wave and use windowed decoder of size \( W_D \) for this part (decode while receive)
- Window latency of order \( W_D + w ( W_D + w - 1 ) \) SPs in window
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Windowed Decoding of Spatially Coupled LDPC Codes

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Windowed Decoding of Spatially Coupled LDPC Codes

Conventional Spatially Coupled LDPC Code
d_v = 5, d_c = 10

\[ I = 1500 \text{ iter.} \]

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- To save latency, we are only interested in left-most portion of wave and use windowed decoder of size \( W_D \) for this part (decode while receive)
- Window latency of order \( W_D + w (W_D + w - 1) \) SPs in window
- Decoding complexity of order \( (W_D + w) \cdot I \) (\( I \): number iterations per window)

Decoding Velocity and Windowed Decoding

Conventional Spatially Coupled LDPC Code  
\[ d_v = 5, \ d_c = 10 \]

• Decoding velocity as displacement of erasure profile per decoding iteration  
  [AStB13], [EM16]

• Decoding velocity \( v \) defined as \( D/I \), where \( I \) is the number of iterations  
  required to advance the profile by \( D \), i.e., here  
  \[ v = D/200 \]


Decoding Velocity and Windowed Decoding

Conventional Spatially Coupled LDPC Code $d_v = 5$, $d_c = 10$

- Windowed decoding only carries out decoding operations on $W_D$ spatial positions that benefit from decoding [ISU+13]
- Complexity of windowed decoding directly linked to the velocity of the profile

Spatially Coupled Codes for High-Throughput Comms.

- **Staircase codes** [SFH+12] now well established in low-complexity, high-throughput optical communications
- Standardized for interoperable communications
- Very good performance with hard-decision decoding
- Spatially-coupled generalized LDPC codes

- Other high-performing spatially coupled codes have been proposed as well
- Example: **Braided BCH codes** presented in [JPN+13]
- Similar performance than staircase codes
- Extra performance gains by using extrinsic decoder requiring more memory

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FPGA-Based Code Evaluation Platform

- **High throughputs & large coding gains** necessary in optical core networks & submarine cables
- **Required BER**: around $0.0000000000001\%$ ($10^{-15}$)
- Maximum 10 bit errors per day at line rate of 100 Gbit/s
- Requirements might become more strict in the future

Virtex-7 based, configurable FPGA emulator platform with windowed decoding
Results of FPGA-Based Code Evaluation ($R = 0.8$)

- Comparison of two different codes
  - Code A: optimized degree dist.
  - Code B: optimized for low floor

Single engine decoder, $I = 1$ iteration of layered decoder [H04]


Results of FPGA-Based Code Evaluation ($R = 0.8$)

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**Single engine decoder, $I = 1$ iteration of layered decoder** [H04]

- Block LDPC Code ($d_v = 3$, $d_c = 15$)

No errors observed
NCG 11.91 dB

**BER**

<table>
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<tr>
<th>Normalized SNR (dB)</th>
<th>BER</th>
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<tr>
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<tr>
<td>2.6</td>
<td>10^{-10}</td>
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<tr>
<td>2.7</td>
<td>10^{-5}</td>
</tr>
<tr>
<td>2.8</td>
<td>10^{-1}</td>
</tr>
<tr>
<td>2.9</td>
<td>10^{-0.5}</td>
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<tr>
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<td>10^{0}</td>
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<tr>
<td>3.2</td>
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Results of FPGA-Based Code Evaluation ($R = 0.8$)

Hybrid Decoder with two engines

H = 

Results of FPGA-Based Code Evaluation ($R = 0.8$)

Hybrid Decoder with two engines

\[ H = \begin{bmatrix} H_1 & H_0 \\ H_1 & H_0 \\ H_1 & H_0 \\ H_1 & H_0 \\ H_1 & H_0 \end{bmatrix} \]

- Engine 1
- Engine 2

Results of FPGA-Based Code Evaluation ($R = 0.8$)

- Net coding gain $12.01$ dB

Results of FPGA-Based Code Evaluation \( (R = 0.8) \)

- **Capacity limit:** 2.06 dB
- **Net coding gain:** 12.01 dB

No errors after \( 6.7 \times 10^{15} \) transmitted bits

**Graph:**
- Block LDPC \( (d_v = 3) \)
- SC-LDPC Code A
- SC-LDPC Code B

**Diagram:**
- Hybrid Decoder with two engines

**Equation:**
\[
H = \begin{bmatrix}
H_1 & H_0 \\
H_1 & H_0 \\
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H_1 & H_0 \\
H_1 & H_0 \\
H_1 & H_0
\end{bmatrix}
\]

**Engine 1**
- Purple

**Engine 2**
- Orange

Results of FPGA-Based Code Evaluation ($R = 0.8$)

**Engine 1**

**Engine 2**

Hybrid Decoder with two engines

- **0.4dB** correspond to **900km reach increase** in trans-pacific cables
- Optical fiber communication systems age (material, lasers, photodiodes) and the SNR will decay over time
- In this case, additional gains increase lifetime/reduce margins of a system
- **More gains** are possible **with higher decoding complexity**
- However, **we want even more gains!**

New: Non-Uniformly Coupled LDPC Code Ensemble
Spatially Coupled LDPC Code with $w = 2$

**Definition**

Connect each edge from variable node at SP $z$ to
- check node at position $z$ with probability $\alpha$ and to
- Check node at position $z + 1$ with probability $1 - \alpha$

Non-Uniformly Coupled LDPC Code Ensemble

Literature Review

- Optimized protographs with implicit non-uniform coupling [MLC15]
- Non-uniform protographs for coded modulation with spatially coupled codes [StB13]
- Non-uniform protographs for improved thresholds and unequal error prot. [JB14]
- Exponential, non-uniform coupling for anytime reliability [NNL15]
- Non-uniform coupling in spatially coupled compressed sensing [KMS+12]
- Rate loss mitigation by extra structure at the boundaries [TKS12], [SP16]

Non-Uniformly Coupled LDPC Code Ensemble

BEC Density Evolution and Rate Loss

- BEC Density evolution for the generalized non-uniformly coupled ensemble

\[ x_z^{(t+1)} = \varepsilon \left( 1 - \sum_{i=0}^{w-1} \nu_i \left( 1 - \sum_{j=0}^{w-1} \nu_j x_z^{(t)} \right) \right)^{d_c-1} d_v^{d_v-1} \]

- In particular, for \( w = 2 \), we have \( \nu = (\alpha, 1 - \alpha) \)

- Rate of the generalized ensemble

\[ R = \left( 1 - \frac{d_v}{d_c} \right) - \frac{d_v}{d_c} \left( w - 1 - \sum_{k=0}^{w-2} \left[ \left( \sum_{i=0}^{k} \nu_i \right)^{d_c} + \left( \sum_{i=k+1}^{w-1} \nu_i \right)^{d_c} \right] \right) \]

- For \( w = 2 \), rate is minimal for \( \alpha = 1/2 \), i.e., non-uniform coupling reduces rate loss
Density Evolution for $\varepsilon = 0.48$, $L = 50$

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$d_v = 5$, $d_c = 10$

$I = 200$ iter.

New: Non-uniformly coupled code with
$d_v = 5$, $d_c = 10$
Single-side convergence
Density Evolution for $\varepsilon = 0.48$, $L = 50$

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**Single-side convergence**
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Single-side convergence

Less iterations required for full convergence!
Non-Uniform Coupling vs. Conventional Uniform Coupling (1)

BP decoding thresholds: SC-LDPC \((d_v, 2d_v, w = 2, \alpha, L = 100)\) over the BEC
Non-Uniform Coupling vs. Conventional Uniform Coupling (1)

BP decoding thresholds: SC-LDPC \( (d_v, 2d_v, w = 2, \alpha, L = 100) \) over the BEC

![Diagram showing the comparison between no coupling and uniform coupling for different values of \( d_v \).]
Non-Uniform Coupling vs. Conventional Uniform Coupling (1)

BP decoding thresholds: SC-LDPC \( (d_v, 2d_v, w = 2, \alpha, L = 100) \) over the BEC

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Increasing
Non-Uniform Coupling vs. Conventional Uniform Coupling (1)

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Increasing \(\alpha^*\): \(\epsilon_{BP, \text{uncoupled}}\) decreases, \(\epsilon_{MAP}\) increases.

Decreasing \(\alpha^*\): \(\epsilon_{BP}(\alpha = 0.5)\) decreases, \(\epsilon_{BP}(\alpha^*)\) increases.
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Decreasing: \(\alpha^*\)

Almost unchanged: \(\epsilon_{BP}\) uncoupled

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Non-Uniform Coupling vs. Conventional Uniform Coupling (1)

BP decoding thresholds: SC-LDPC \((d_v, 2d_v, w = 2, \alpha, L = 100)\) over the BEC

<table>
<thead>
<tr>
<th>(d_v)</th>
<th>(\alpha^*)</th>
<th>(\varepsilon_{BP}) uncoupled</th>
<th>(\varepsilon_{MAP})</th>
<th>(\varepsilon_{BP}(\alpha = 0.5))</th>
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Non-Uniform Coupling vs. Conventional Uniform Coupling (2)

Complexity of Decoding (i.e., Number of Iterations)

Decoding speed contour plots for the random SC-LDPC\( (d_v, 2d_v, w = 2, \alpha, L = 100) \) ensemble.

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Same velocity with higher variable node degree \(\rightarrow\) lower error floors

Uniform coupling
What Happens with Optimized SC-LDPC Codes ($R = 0.8$)

- FPGA simulation results of QC versions of these codes
- Degraded performance of optimized, unequally coupled codes under windowed decoding [SSA+16]
- Performance does not correspond to predicted threshold
- What is happening?

Windowed Decoder Stall
Exemplary Error Patterns AFTER Decoding

- In rare cases, decoder gets stuck
- Subsequent spatial positions are also stuck
- *Burst-like error pattern*
Observations Inside the Decoding Window

- $p_{\text{win}} \in (1, N_W)$ denotes the window position
- Decoder gets stuck around $p_{\text{win}} = 37$
- Leftmost position(s) needs to be error-free before decoding

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\[ \cdot 10^{-2} \]

\[ \begin{array}{c|c|c|c|c|c|c|c} \hline
\text{BER} & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
\text{Spatial position within decoding window} & & & & & & & \\
\hline
\end{array} \]

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Decoding Window Loses Track of Decoding Wave

With probability $p_B$, a burst-situation occurs in a codeword

Decoding Window Loses Track of Decoding Wave

- Estimated $p_B$ for the codes used in previous simulation:

<table>
<thead>
<tr>
<th>$E_b/N_0$ (dB)</th>
<th>$p_B$ for $L = 24$</th>
<th>$p_B$ for $L = 99$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.84</td>
<td>$1.0 \cdot 10^{-3}$</td>
<td>$5.6 \cdot 10^{-2}$</td>
</tr>
<tr>
<td>2.87</td>
<td>$1.6 \cdot 10^{-4}$</td>
<td>$6.3 \cdot 10^{-4}$</td>
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<tr>
<td>2.90</td>
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<tr>
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- With probability $p_B$, residual errors occur.

References:


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• **Idea 3**: *Stall Detection*
  • React when stall is about to happen

Decoder Stall Detection

Decoder stall detection

- **Variant A**: Stall detection based on fulfilled parity checks (HD)
- **Variant B**: Stall detection based on estimated BER (SD)
  - Estimate BER within SP inside windowed decoder as \[ \text{[HIS00]} \]
  \[ \text{BER}_i = \frac{1}{M} \sum_{k=1}^{M} \frac{1}{1 + \exp(|L_{i,k}|)} \]
  - Use \( \text{BER}_i \) thresholds to estimate position of wave inside decoder
  - React by carrying out more iterations or shifting window (Strat. A, B, C)

Strategy A - Adaptive Iterations Decoder

- Stall detected: Increase number of iterations
- No stall present: Shift window after minimum number of iterations
Strategy A - Adaptive Iterations Decoder

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- After $I$ iterations, continue with next window
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- Adaptive shifting can be implemented using some simple buffering and control [SL14]


Conclusions & Outlook

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  • Decoding threshold
  • Rate loss
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- Feasible coding scheme promising additional gains, but need HW architectures
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- State-of-the-art FEC schemes proposed for practical implementation
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- The best performing schemes are spatially coupled codes
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